# *A Correction of the* Theory *of the Motion of the* Satelite *of* Saturn, *by that Ingenious* Aſtronomer Mr. Edmund Hally.

*SIR*,

I here ſend you an *Aſtronomical account* of the moſt remote of all the *Planets* of our *Vortex*, and withal of the moſt inconſiderable, I mean of the *Satellite* of *Saturn*, diſcovered in the year 1655 by *Monſieur Chriſtian Hugens* of *Zulichem*, who in that accurate Treatiſe of his *Syſtema Saturnium*, from *pag.* 25 to 34 gives us the *Theory* of its *Motion* as well as the ſhortneſs of the interval of time between his *Obſervations* would admit; and ſince him I know none that have gone about to improve the ſaid *Theory*.

The late *Conjunctions* of *Jupiter* and *Saturn* giving me frequent occaſions of viewing them both, with a *Teleſcope* that I have of about 24 foot, and pretty good of that length, I eaſily remarqued this *Satallite* of *Saturn*, and haveing found it, in a convenient poſition to determine its place, I percieved that *Hugens*'s numbers were conſiderably run out, and about 15 *degrees* in twenty years too ſwift; this made me reſolve more nicely to enquire into its *period*; and accordingly I waited till I had gotten a competent number of *Obſervations*, the moſt conſiderable whereof are theſe.

1682. *November* $13^{\circ}$ $13^{h}$ $00'$ PM. the *Satellite* appeared on the *North* ſide of *Saturn*, and a *perpendicular* let fall from it on the tranſverſs *diameter* of the *Ring*, fell upon the middle of the dark ſpace of the following *Anſa*; and the ſame night $19^{h}$ $00'$ it had paſt the *Conjunction*, and the *perpendicular* fell exactly on the *Weſtern* edg of the *Globe* of *Saturn*, as in *Tab.* 2. *Figure* 8. The *Northern* Latitude, and retrograde motion, made it evident that the *Satellite* was then in *Perigæo*.

Again *November* 21°. 16$^{h}$. 15′. this *Satellite* of *Saturn* was on his *South* ſide, the *perpendicular* on the line of the *Anſæ* fell on the middle of the dark ſpace of the *Weſtern Anſæ*, and the ſame night 19$^{h}$ 00′ the *perpendicular* fell preciſly on the *Center* of *Saturn*, and the diſtance therefrom was ſomewhat leſs than one *diameter* of the *Ring* (as in *Fig. 9.)* by this it was evident that the *Satellite* was in *Apogæo.*

I obſerved it in *Apogæo* again on the 24th of *January* 1683, at 8$^{h}$. 00′ PM. the *perpendicular* on the line of the *Anſæ* fell exactly on the *Weſtern* limb of the *Globe* of *Saturn*, and at 9$^{h}$ 30′ PM. the ſaid *perpendicular* fell within the *Globe* more than half way to the *Center*, and the diſtance from the line of the *Anſæ* towards the *South* ſeemed much about one *diameter* of the *Ring*. *Fig.* 10.

Laſtly, *February* 9°. 1683. 8$^{h}$ 10′ PM. it was again in *Apogæo*, and I could by no means diſcern towards which ſide it enclin'd moſt, nor whether the tranſverſe *diameter* of the *Ring*, or the diſtance of the *Satellite* therefrom were the greater; ſo that at that time it was preciſly *Apogæon. Fig.* 11.

To compare with theſe, I choſe two out of thoſe of *Monſieur Hugens*, which ſeemed the moſt to be confided in; the firſt made 1659, *March* 14° *ſt. n.* 12$^{h}$ 00′. at the *Hague*; when the *Satellite* appeared about one *diameter* of the *Ring* under *Saturn*, but it was gone ſo farr to the *Weſtward*, that he concluded, that about four hours before, or 7$^{h}$. 40′. at *London*, it had been in *Perigæo. Fig.* 12.

Again *March* 22°. 1659. 10$^{h}$ 45′. the *Satellite* was a whole *diameter* above the line of the *Anſæ*, and the *perpendicular* thereon fell nearly upon the extremity of the *Eaſtern Anſæ*. See *Fig.* 6.

I could wiſh that we had ſome intermediate *Obſervations*, but there are none extant that I can hear of; ſo I proceed to the ſearch of this *Satellit*'s period.

By the firſt of my *Obſervations* it appears that the *Satellite* was in *Perigæo* 1682 *November* 13°. 17$^{h}$ 00′. *circiter*, at which time *Saturn* was 3$^{s}$. 21°. 39′ from the firſt Star of *Aries*, in the *Ecliptick*, but the *Earth* reduced to *Saturns Equinoctial*, and the *Satellite* was 9$^{s}$. 23° 46′. a 1$^{a}$ * ♈. And *March* 4° 1659. 7$^{h}$. 40′. *Saturns* place in the *Ecliptick* was 6$^{s}$. 0°. 41′, but the *Earth* reduced and conſequently

conſequently the *Sattllite* in 11$^{s}$. 28°. 18′ *à prima Stella Arietis.* The interval of time is 8655 daies, 9 hours, 20 minutes; in which the *Satellite* had made a certain number of *Revolutions* to the fixt Stars, and beſides 9$^{s}$. 25°. 28′, or 295 *degrees*, 28 *min.* whoſe *Complement* to a Circle 64°. 32′, is 2 daies, 20$^{h}$, 36′ motion of the *Satellite*, according to *Hugens.* So that 8658 daies, 5$^{h}$, 56′ or 12467876 minutes of time, is the time of ſome number of intire *Revolutions*; and dividing that interval by 15 daies 22 hours, 39 minutes, or 22959 min. (the *Period* of *Hugens*) the *Quotient* 543 ſhewes the number of *Revolutions*; and again dividing 12467876 min. by 543, the 22961 ½ min. or 15 daies, 22$^{h}$. 41′. 6″ appears to be the true time of this *Satellit's Period.* Hence the *diurnal motion* will be 22°. 34′. 38″. 18‴, and the *Annual* beſides 22 *Revolutions* 10$^{s}$. 20°. 43′. Having made *Tables* to this Period, I found that in the *Apògæon* Obſervation of *Hugens* the *Satellite* was above 3 degrees faſter than by my *calculus*, and that in the three other Obſervations of my own being likewiſe in the ſuperior part it was about 2½ *degrees* ſlower than by the ſame *Calculation.* Now 'tis evident that theſe differences muſt ariſe from ſome *Eccentricity* in the *Orbite* of this *Satellite*, and that in *March* 1659, the *Apocronion* (as I may call it,) was ſomewhere in the *Oriental Semicircle*, and that in *November* 1682 it was in the *Weſtern Semicircle*, and ſuppoſing the *Apocronion* fixt, it muſt neceſſarily be between 9$^{s}$. 23° 46′. and 11$^{s}$. 28°. 18′. a 1$^{a}$ * ♈ that being the common part between thoſe two *Semicircles*: and becauſe the difference was greater in *Hugen's* Obſervation than in *Mine*, 'twill follow that the *Linea Apſidum*, or *Apocronion*, ſhould be nearer to 9$^{s}$. 23°. 46′. than to 11$^{s}$. 28°. 18′. I will ſuppoſe 10$^{s}$. 22°. 00′ *a prima Stella Arietis*, (which happens to be alſo the place of *Saturn Equinox*,) and the greateſt *æquation* about 2½ degrees. Upon the ſcore of this *inequality* the *mean* motion of the *Satellite* will be found about 2°. 45′ ſlower in 23½ *years*, or 7 min. in a *year*, whence I ſtate the *Annual* motion 10$^{s}$. 20°. 36′ above 22 *Revolutions*, and the correct *Epocha* for the laſt day of *December* 1682 at Noon in the *Meridian* of *London* 9$^{s}$. 10°. 15′. *a* 1$^{a}$ * ♈, from which Elements I compoſe the following *Table.*

# *Tabula Motus Medii Satellitis Saturnii,*

*ab* Hugenio *inventi, a prima* * ♈.

| *Ann. Chriſt. Curr.* | *Epochæ* s. o. ' |
|---|---|
| 1641 | 8. 29. 17. |
| 1661 | 10. 14. 10. |
| 1681 | 11. 29. 3. |
| 1682 | 10. 19. 39. |
| 1683 | 9. 10. 15. |
| 1684 | 8. 00. 51. |
| 1685 | 7. 14. 2. |

| *Menſ. Ann. Com.* | *Mot. Med.* s. o. ' |
|---|---|
| *Jan.* | 0. 0. 0. |
| *Feb.* | 11. 9. 53 |
| *Mart.* | 8. 12. 2. |
| *April.* | 7. 21. 56 |
| *Maii* | 6. 9. 14. |
| *Junii* | 5. 19. 7. |
| *Julii* | 4. 6. 26. |
| *Aug.* | 3. 16. 19 |
| *Sept.* | 2. 26. 12. |
| *Octob.* | 1. 13. 31. |
| *Novem.* | 0. 23. 24. |
| *Decem.* | 11. 10. 43. |

| *Annis* | *Mot. Med.* s. o. ' |
|---|---|
| 1 | 10. 20. 36 |
| 2 | 9. 11. 12 |
| 3 | 8. 1. 48 |
| 4 | 7. 14. 59 |
| 5 | 6. 5. 35 |
| 6 | 4. 26. 11 |
| 7 | 3. 16. 47 |
| 8 | 2. 26. 57 |
| 9 | 1. 20. 23 |
| 10 | 0. 11. 9 |
| 11 | 11. 1. 45 |
| 12 | 10. 14. 56 |
| 13 | 9. 5. 32 |
| 14 | 7. 26. 8 |
| 15 | 6. 16. 44 |
| 16 | 5. 29. 54 |
| 17 | 4. 20. 30 |
| 18 | 3. 11. 6 |
| 19 | 2. 1. 42 |
| 20 | 1. 14. 53 |

| *Diebus* | *Mot. Med.* s. o. ' |
|---|---|
| 1 | 0. 22. 35 |
| 2 | 1. 15. 9 |
| 3 | 2. 7. 44 |
| 4 | 3. 0. 18 |
| 5 | 3. 22. 53 |
| 6 | 4. 15. 28 |
| 7 | 5. 8. 2 |
| 8 | 6. 0. 37 |
| 9 | 6. 23. 12 |
| 10 | 7. 15. 46 |
| 11 | 8. 8. 21 |
| 12 | 9. 0. 55 |
| 13 | 9. 23. 30 |
| 14 | 10. 16. 5 |
| 15 | 11. 8. 39 |
| 16 | 0. 1. 14 |
| 17 | 0. 23. 48 |
| 18 | 1. 16. 23 |
| 19 | 2. 8. 58 |
| 20 | 3. 1. 32 |
| 21 | 3. 34. 7 |
| 22 | 4. 16. 42 |
| 23 | 5. 9. 16 |
| 24 | 6. 1. 51 |
| 25 | 6. 24. 25 |
| 26 | 7. 17. 00 |
| 27 | 8. 9. 35 |
| 28 | 9. 2. 9 |
| 29 | 9. 24. 44 |
| 30 | 10. 17. 18 |
| 31 | 11. 9. 53 |
| 32 | 0. 2. 28 |

| *H.* *M.* | *Mot. Med.* o. ' / '. ". | *M.* | *Mot. Med.* '. ". |
|---|---|---|---|
| 1 | 0. 56 | 31 | 29. 10 |
| 2 | 1. 53 | 32 | 30. 6 |
| 3 | 2. 49 | 33 | 31. 3 |
| 4 | 3. 46 | 34 | 31. 59 |
| 5 | 4. 42 | 35 | 32. 55 |
| 6 | 5. 39 | 36 | 33. 52 |
| 7 | 6. 35 | 37 | 34. 48 |
| 8 | 7. 32 | 38 | 35. 45 |
| 9 | 8. 28 | 39 | 36. 41 |
| 10 | 9. 24 | 40 | 37. 38 |
| 11 | 10. 21 | 41 | 38. 34 |
| 12 | 11. 17 | 42 | 39. 31 |
| 13 | 12. 14 | 43 | 40. 27 |
| 14 | 13. 10 | 44 | 41. 24 |
| 15 | 14. 7 | 45 | 42. 20 |
| 16 | 15. 3 | 46 | 43. 17 |
| 17 | 16. 0 | 47 | 44. 13 |
| 18 | 16. 56 | 48 | 45. 10 |
| 19 | 17. 52 | 49 | 46. 6 |
| 20 | 18. 49 | 50 | 47. 3 |
| 21 | 19. 45 | 51 | 47. 59 |
| 22 | 20. 42 | 52 | 48. 56 |
| 23 | 21. 38 | 53 | 49. 52 |
| 24 | 22. 35 | 54 | 50. 49 |
| 25 | 23. 31 | 55 | 51. 45 |
| 26 | 24. 27 | 56 | 52. 42 |
| 27 | 25. 24 | 57 | 53. 38 |
| 28 | 26. 20 | 58 | 54. 35 |
| 29 | 27. 17 | 59 | 55. 31 |
| 30 | 28. 13 | 60 | 56. 27 |

*In Anno Biſſextili poſt Februarium adde unum diem, motumq; ei competentem.*

I here ſuppoſe the *Linea Apſidum* fixt, as having no arguments from Obſervation to prove the contrary, tho it be very probable that as the *Apogæon* of our *Moon* has a motion about the *Earth* in about 9 years, ſo that [illegible] this *Satellite* ought to have about *Saturn*, but with a much longer *Period*, which future Obſervation may diſcover.

The diſtance of this *Satellite* from the *Center* of *Saturn* ſeems to be much about 4 *Diamiters* of the *Ring*, or 9 of the *Globe*, and the plane wherein it moves very little or no thing differing from that of the *Ring*, that is to ſay, interſecting the *Orb* of *Saturn* $4^s.\,22^\circ$. and $10^s.\,22^\circ.\,41'$. * ♈, with an *Angle* of 23½ *degrees*, ſo as to be nearly Parallel to the *Earths Equator*; whence the *Latitude* of the *Apogæon* Semicicle from $4^s.\,22^\circ$. to $10^s.\,22^\circ$ of *Saturns Longitude*, will be *Northern*, and of the other Semicircles *Southern*; and the contrary in the other half of *Saturns Longitude*, to wit, from $10^s$, $22^\circ$. to $4^s.\,22^\circ$ of his diſtance from the firſt Star of ♈.

It follows now to ſhew how by the help of this *Table* to compute the place of this *Satellite*, to any time required.

Firſt we muſt have the true *Longitude* of *Saturn* from the *Earth*, and numbred from the firſt Star of ♈, (or rather the place of the *Earth* viewed from *Saturn* together with its *Latitude* from the *Orb* of *Saturn*, but that being never fully ⅓ of a *degree* we neglect it as a *nicety*) and therefrom *ſubſtract* $10^s.\,22^\circ$. there remains the diſtance of *Saturn* from this *Æquinoctial* point, with which diſtance as with the *Longitude* of the *Sun*, take out the *Right Aſcenſion* and *Declination* thereto (23½ degrees being the *obliquity* common to both) and to the *Right Aſcenſion* adding $10^s.\,22^\circ$. the ſumm ſhall be the *Longitude* of the *Satelli's Apogæon*. Then ſay, As *Radius* to *ſine* of the *Declination*, ſo 8 to the greateſt *Latitude* in *Apogæo*, or *Perigæo* in the parts of the *ſemidiamiter* of the *Ring*.

Next collect the *middle motion* of the *Satellite*, and from it *ſubſtract* $10^s.\,22^\circ$; the *remainder* ſhall be the *mean Anomaly*, with which in the *Table* of the *Moons primary Equation*; take out the *Equation* anſwering thereto, and the half thereof *added* or *ſubſtracted* to or from the *middle motion*, according to the *Title*, gives the *true motion* of the *Satellite*, from which *ſubſtract* the *Apogæon*, and if the *remainder* be more than 6 *Signs*, the *Satellite* is *Occidental*; if leſs *Oriental*, and as *Radius* to *Sine* of the *remainder*, ſo 8 to the *Semidiamiters* of the *Ring*; or 18, to the *Semidiameters* of the *Globe*; that the *Satellite* is to the *Eaſtward* or *Weſtward* of the *center* of *Saturn*, according to the afore-going *Precept*.

Laſtly, As *Radius*, to *Co-ſine* of the ſaid *remainder*, ſo is the greateſt *Latitude* from the *line* of the *Anſæ*, to the *Latitude* ſought.

ariſe-

Here Note, that I purpoſely neglect the *inequality* of the diſtance, ariſing from the *Eccentricity*, as being too ſmall to be any way obſervable.

Laſtly to clear all difficulties that may ariſe to them that are but little *verſed* in this ſort of *Calculation*, I have added Two *Examples* of the work, that where the *Precept* may ſeem obſcure it may be thereby illuſtrated.

*Anno* 1657 *Maii* 19 *ſt. n. Hugens* Obſerved the *Satellite* very near to *Saturn* on the *Weſtern* ſide, and very little above the *line* of the *Anſæ*. I ſuppoſe this about $10^{h}$. *p. m. Anno* 1658 *Martii* 11°. $10^{h}$ *ſt. n.* he Obſerved it again, and ſaith of it, *difficilè conſpiciebatur, quippe propinquius admodum* Saturno, *Orientem ſpectabat, eratq; Anſarum lineâ aliquanto inferior. & quaſi ſub* Saturno *tranſiturus.* Let us now Calculate to theſe two times.

| 1657 *Maii* 9°. | $9^{h}$. 40′. | *Londini* | |
|---|---|---|---|
| *Saturni Locus* | ♍. | 28 . | 57′ |
| ♄ *a* $1^{a}$ * ♈ | $5^{s}$. | 0. | 32 |
| *Æquinoct. ſub.* | 10. | 22. | 00 |
| ♄ *ab Æquinoct.* | 6. | 8. | 32 |
| *Aſcen. Recta.* --- | 6. | 7. | 50 |
| *Apogæon.* | 4. | 29. | 50 |
| *Declin. Auſt.* | | 3. | 23 |

| | *Med.* | *Mot.* | *Satel.* |
|---|---|---|---|
| 1641. | $8^{s}$. | 29°. | 17′ |
| 16. | 5. | 29. | 54 |
| *Maii* | 6. | 9. | 14 |
| 9. | 6. | 23. | 12. |
| $9^{h}$. 40′ | | 9. | 5 |
| *Long. Med. Satel.* | 4. | 10. | 42 |
| *Apocron.* | 10. | 22. | 00 |
| *Anomalia* | 5. | 18. | 42 |
| *Æquatio ſub.* | | | 31 |
| *Long. Ver. Satel.* | 4. | 10. | 11 |
| *Apogæon* | 4. | 29. | 50 |
| *Reſiduum* | 11. | 10. | 21 |
| *h. e. ante Apogæum* | | 19. | 39 |

ergo $2\frac{7}{10}$ *Semid. Annuli ad occaſum* & $\frac{9}{20}$ *ad Boream.*

| 1658. *Martii* | 1° | | $10^{h}$ |
|---|---|---|---|
| *Saturni Locus* | ♎ | 16°. | 25′ |
| ♄ *a* $1^{a}$ * ♈ | $5^{s}$ | 17°. | 59 |
| *Æquinoctium* | 10 | 22. | 00 |
| ♄ *ab Æquin.* | 6. | 25. | 59 |
| *Aſcen. Recta.* | 6. | 24. | 5 |
| *Apogæon* | 5. | 16. | 5. |
| *Declin. Auſt.* | | 10. | 4 |

| | *Med.* | *Mot.* | *Satel.* |
|---|---|---|---|
| 1641. | $8^{s}$. | 29°. | 17′ |
| 17. | 4. | 20. | 30½ |
| *Martii.* | 8. | 12. | 2½ |
| $1^{d}$. | 0 | 22. | 34½ |
| $10^{h}$ | | 9. | 24½ |
| *Long. Med. Satel.* | 11. | 3. | 49 |
| *Apocron.* | 10. | 22. | 00 |
| *Anomalia* | 0. | 11. | 49 |
| *Æquatio ſub.* | | 0. | 30 |
| *Long. Ver. Satel.* | 11. | 3. | 19 |
| *Apogæon* | 5. | 1 . | 5 |
| *Reſiduum* | 5. | 17. | 14 |
| *h. e. ante Perigæon* | | 12. | 46 |

*hinc provenit* $1\frac{22}{100}$ *Semid. Ann. ad ortum,* & $1\frac{36}{100}$ *ad Auſtrum ſive infra.*

In each agreeing exactly with the Deſcription and Figure of Monſieur *Hugens.*

I here call the *Plane* of this *Satellit's Orb*, which hitherto I ſuppoſe the ſame with that of the *Ring*, *Saturns Equinoctial*, not that any diſcovery hath been able to prove that the *Axis* of that *Globe* is at *right Angles* thereto,but becauſe it has pleaſed Mr. *Hugens* to call it ſo, and likewiſe becauſe it is ſo nearly *Parallel* to our *Globes Equinoctial*; Nevertheleſs to ſpeak my Opinion, I believe that the *Axis* is inclined, and that not a little, to the *Plane* of the *Ring*; for as the reflection of the *Suns* light from the *Ring* is a great convenience to that *Hemiſphere* of *Saturn*, which beholds its illuminate ſide; ſo the other *Hemiſphere* is very much incommoded by the ſhadow of the *Ring*, which for many *Months*, and in ſome *Parallels* for ſeveral *Years*, occaſions a continual Night by the interception of the *Suns* beams, which is a conſequence that demonſtratively follows the poſition of the *Ring* in the *plane* of *Saturns Equator*. Now this great inconvenience would be in ſome meaſure relieved by the oblique poſition of the *Axis*, for then the *Parallels* of *Latitude* interſecting the *plane* of the *Ring*, many and in moſt caſes all of them, might for ſome time in every *Diurnal revolution* of the *Globe* free themſelves from this *Eclipſe*, which otherwiſe were ſufficient to render this *Globe* of *Saturn* unfit for any ſettled habitation; but this is but conjecture.

The other Two *Satellites* of *Saturn* diſcovered by Signor *Caſſini* at *Paris Anno* 1672 and 1673, I muſt confeſs I could never yet ſee. I have been told that they diſappear for about $\frac{2}{3}$ of *Saturns revolution*, and were only to be ſeen when the *Anſæ* were very *ſmall*, it being ſuppoſed that the light which proceeds from the *Anſæ* when conſiderably opened might hide theſe *Satellits*. In the year 1685 when the *Anſæ* will be quite vaniſhed, will be a proper time to look after them, that ſo we may bring their Motion to Rule, and know where to find them, for want of which knowledge 'tis likely they are at preſent not to be found.

*Philoſoph. Tranſact. Numb. 145.* Tab. 2.

Fig. 1.

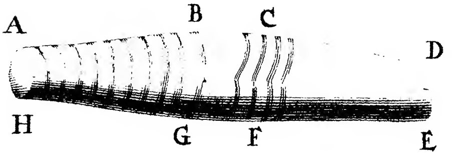


Fig. 2.

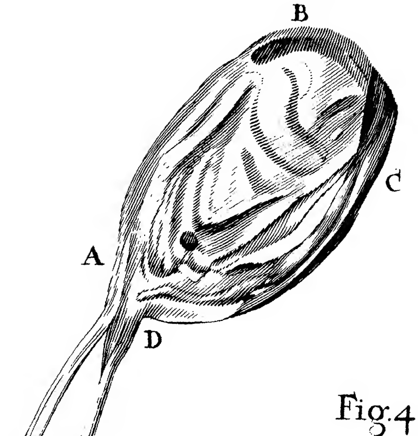


Fig. 3.

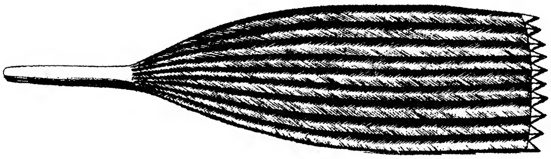


Fig. 4.

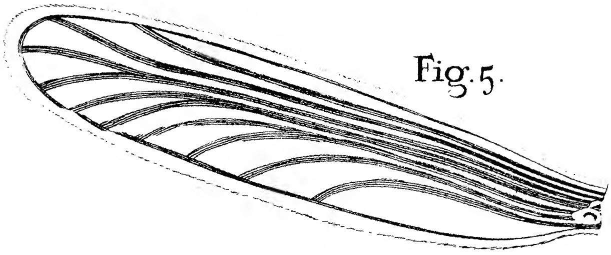


Fig. 5.

Fig. 6.

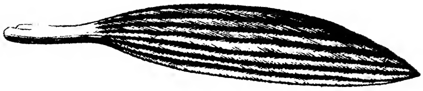
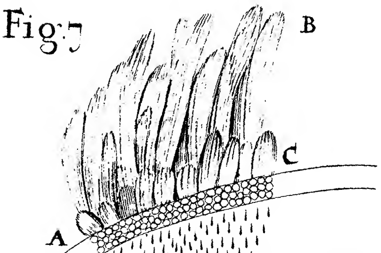


Fig. 7



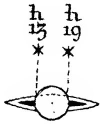


Fig. 8.

Fig. 9.

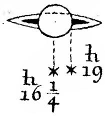


Fig. 10

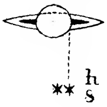


Fig. 11.

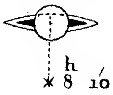
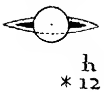


Fig. 12.



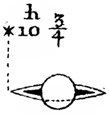


Fig. 13.

Fig. 14.